AINM NAME | ÁBHAR SUBJECT
MÚINTEOIR TEACHER

3rd yr Christmas 2012 Solutions

Q1(a)

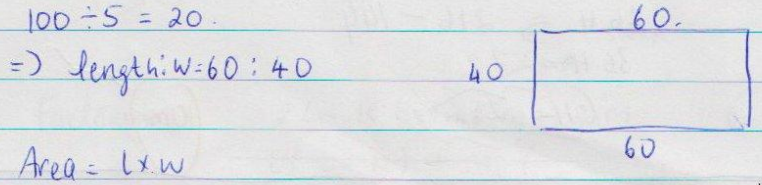
Perimeter = $2L + 2W = 200$

$L + W = 100$

(Ratio 3:2, $3+2=5$)

$100 \div 5 = 20$

$\Rightarrow$ length : W = 60 : 40

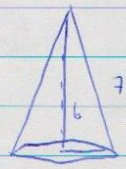


Area = $L \times W$

$= 60 \times 40$

Area = $2400\ cm^2$ (10)m

(b)(i)



pythagoras

$(7.5)^2 = (6)^2 + R^2$

$56.25 = 36 + R^2$

$20.25 = R^2$

$\sqrt{20.25} = R$

$4.5 = R$

Radius = 4.5

(50) Q1 Total

(10)m

(ii) Total Surface Area = Sides + Top Circle

$\pi R \cdot L + \pi R^2$

$= \pi(4.5)(7.5) + \pi(4.5)^2$

$= 54\pi$

$= 169.646$ (169.600

(10m)

(Ci) Volume of container

Vol of cylinder + Vol of hemisphere

$\pi R^2 H + \frac{2\pi R^3}{3}$

$\pi(6)^2(14) + \frac{2}{3}\pi(6)^3$

$504\pi + 144\pi$

$648\pi\ cm^3$

(10m)

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Volume of cylinder + volume of hemisphere = 216π (1/3 volume)

$\pi R^2 H + \frac{2}{3}\pi R^3 = 216\pi$

$(6)^2(H) + \frac{2}{3}(6)^3 = 216$

$36H = 216 - 144$

$36H = 72$

$H = 2$ cm

10m

Q2 (a)

Sunday = x°

Monday = $x+3$

Tuesday = $x+3+3 = x+6$

Wed = $x+6-4 = x+2$

Thur = $x+2-4 = x-2$

Fri = $x-2-4 = x-6$ friday.

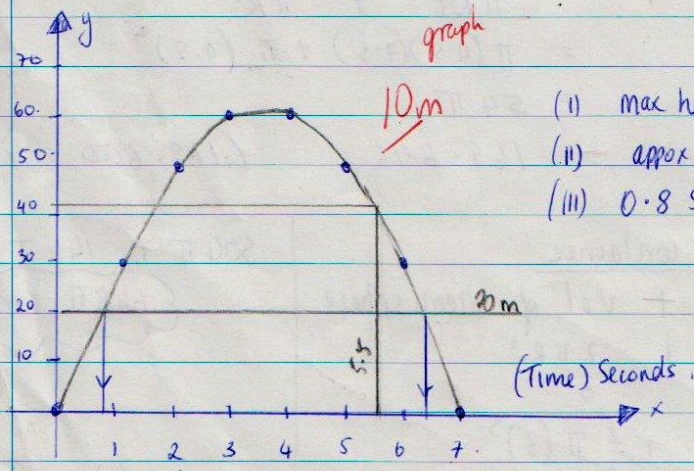
10m

(b)

$x=0$	$35(0)-5(0)^2 = 0$	$x=3$	$35(3)-5(3)^2 = 60$
$x=1$	$35(1)-5(1)^2 = 30$	$x=4$	$35(4)-5(4)^2 = 60$
$x=2$	$35(2)-5(2)^2 = 50$	$x=5$	$35(5)-5(5)^2 = 50$
		$x=6$	$35(6)-5(6)^2 = 30$
		$x=7$	$35(7)-5(7)^2 = 0$

10 points

Points (0,0) (1,30) (2,50) (3,60) (4,60) (5,50) (6,30) (7,0)



graph 10m

(i) max height = 60.5 m 5/

(ii) appox 43 m 5/

(iii) 0.8 seconds, 6.4 seconds 5/

5/

50m Q2

Q3. $v^2 = u^2 + 2as$ (Aim: $s = \ldots$)

(a)

$v^2 - u^2 = 2as$

$\frac{v^2 - u^2}{2a} = s$

10m

(b) (i) Factorise $2L - KL + Km - 2M$ (HCF Rule)

~~$L(2-K) + M($~~

$2L - KL - 2M + Km$ (needed to be rearranged to get Signs Right)

$L(2-K) - m(2-K)$

$(L-m)(2-K)$

5m

(ii) Factorise: $6x^2 - 19x + 10$ GN (60)

$6x^2 - 4x - 15x + 10$ 160

$2x(3x-2) - 5(3x-2)$ 230

$(2x-5)(3x-2)$ -4-15

3m

(iii) $17x - 5x^2$ [HCF]

$x(17 - 5x)$

5m

(iv) $(2x-1)^2$ $(x-1)^2$

$2x(2x-1) - 1(2x-1)$ $x(x-1) - 1(x-1)$

$4x^2 - 2x - 2x + 1$ $x^2 - 1x - 1x + 1$

$4x^2 - 4x + 1$ $x^2 - 2x + 1$

$[4x^2 - 4x + 1] - [x^2 - 2x + 1]$

$4x^2 - 4x + 1 - x^2 + 2x - 1$

$\boxed{3x^2 - 2x}$

5m

(c) (i) $\frac{1}{x-1} + \frac{1}{x+1}$ Common denominator $(x-1)(x+1)$

10m

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$4x^2 - 4x + 1$ $x^2 - 2x + 1$

$[4x^2 - 4x + 1] - [x^2 - 2x + 1]$

$4x^2 - 4x + 1 - x^2 + 2x - 1$

$\boxed{3x^2 - 2x}$

5m

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10m

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$= \frac{1(x+1)}{(x-1)(x+1)} + \frac{1(x-1)}{(x+1)(x-1)}$

$= \frac{x+1+x-1}{(x-1)(x+1)} = \frac{2x}{(x-1)(x+1)}$

(ii) hence $\frac{1}{x-1} + \frac{1}{x+1} = 3.$

$\frac{2x}{(x-1)(x+1)} \times 1 = \frac{3}{1}$

50m Q3

$2x(1) = 3(x-1)(x+1).$

$2x = 3(x(x+1) - 1(x+1))$

$2x = 3(x^2 - 1)$

move $\rightarrow 2x = 3x^2 - 3.$

$3x^2 - 2x - 3 = 0$ $\quad a=3 \quad b=-2 \quad c=-3$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(3)(-3)}}{2(3)}$ 10m

$x = \frac{2 \pm \sqrt{40}}{6}$ $\quad x = \frac{1 \pm \sqrt{10}}{3}.$

50m Q4

Q4 $f(x) = 1 - 3x.$

(i) $F(-2) = 1 - 3(-2) = 1 + 6 = 7.$ 5m

$g(5) = 1 - x^2 = 1 - (5)^2 = -24$ 5m

(ii) $f(x+1) = 1 - 3x = 1 - 3(x+1) = 1 - 3x - 3$ $\quad -3x - 2$ 5m

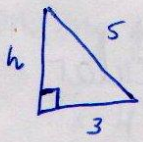
(iii) $f(x+1) = f(-2) + g(5)$

$-3x - 2 = 7 + (-24)$

$-3x - 2 = -17 \qquad -3x = -15 \qquad x = 5$ 5m

Q4

(b) (i)



$=$

$L^2 = h^2 + R^2$

$5^2 = (h)^2 + (3)^2$

$25 = h^2 + 9$

$25 - 9 = h^2$

$16 = h^2$

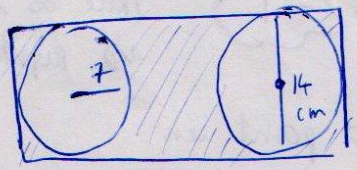
$\boxed{h = 4}$

pythagoras to find missing dimension in a triangle

5m

(ii) $\frac{1}{3}\pi R^2 H = \frac{1}{3}(\pi)(3)^2(4) = \boxed{12\pi}$ 5m

(c)



$14 = L$.

16π

(i) $7 + 7 = 14$ (diameter) 5m

(ii) Shaded Region =

Rectangle − Circles

$L \times W - \pi R^2 \times 2$

$(16\pi)(14) - 2(\pi)(7)^2$

$224\pi - 98\pi$

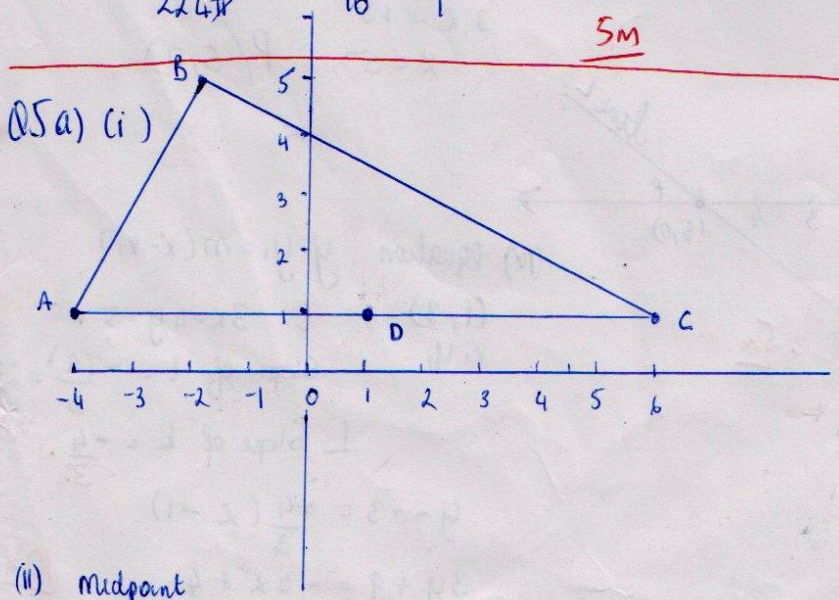
$\boxed{126\pi}$ 10m

(iii) $\frac{\text{Shaded Region}}{\text{Total area}}$

$= \frac{126\pi}{224\pi} = \frac{9}{16} \times \frac{100}{1} = 56.25\%$

5m

Q5a) (i)



5m

(60) Q5

(ii) Midpoint

A $(-4,1)$ $x_1 y_1$ C $(6,1)$ $x_2 y_2$ $= \left(\frac{-4+6}{2}, \frac{1+1}{2}\right) = \left(\frac{2}{2}, \frac{2}{2}\right) = (1,1)$ midpoint D.

5m

(iii) Circle drawn. 5m

(iv) Show ∠ABC is Right angle ⟹ Perpendicular

Slope 1 × Slope 2 = −1

Slope of AB $(-4,1)\ (-2,5)$ = $\frac{y_2-y_1}{x_2-x_1} = \frac{5-1}{-2--4} = \frac{4}{2} = 2.$

$x_1 y_1 \quad x_2 y_2$

Slope of BC $(-2,5)\ (6,1) = \frac{1-5}{6--2} = -\frac{4}{8} = -\frac{1}{2}$

$x_1 y_1 \quad x_2 y_2$

10m

Slope 1 × Slope 2 = $(2) \times (-\frac{1}{2}) = -1$ True so its right angled.

(b) (i) $3x - 4y - 15 = 0$ · sub point in $(1,-3)$

$x \quad y$

$3(1) - 4(-3) - 15 = 0$

$3 + 12 - 15 = 0$

$0 = 0$ true

5m

(ii) on x axis $y = 0$

$3x - 4y - 15 = 0$

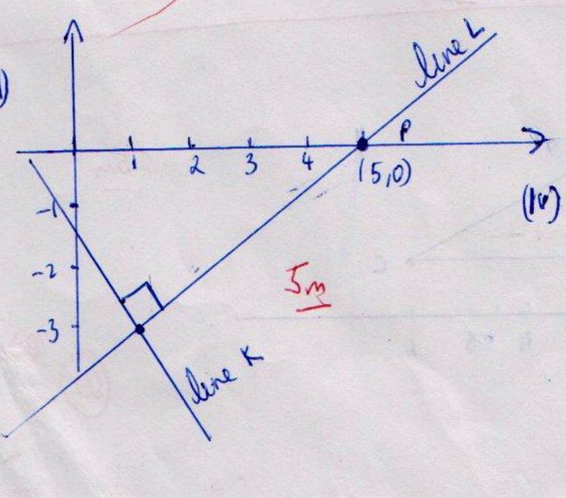
$3x - 4(0) - 15 = 0$

$3x = 15$

$x = 5$ $P(5,0)$

5m

(iii)



(iv) equation $y - y_1 = m(x - x_1)$

$(1,-3)$

$x_1 y_1$

L: $3x - 4y - 5 = 0$

Slope of L = $-\frac{(a)}{b} = \frac{-3}{-4}$

⊥ Slope of K = $\frac{-4}{3}$.

$y - -3 = \frac{-4}{3}(x - 1)$

$3y + 9 = -4x + 4$

$\boxed{4x + 3y + 5 = 0}$ line K.

10m

(v) on my graph $(1, -3.2)$ 5m

(vi) $3x - 4y - 15 = 0 \quad (\times 3)$

$4x + 3y + 5 = 0 \quad (\times 4)$

$9x - 12y - 45 = 0$

$16x + 12y + 20 = 0$

$25x - 25 = 0$

$x = 1$

$3x - 4y - 15 = 0$

$3(1) - 4y - 15 = 0$

$-4y - 12 = 0$

$-4y = 12$

$4y = -12$

$y = -3$

$\boxed{(1, -3)}$ is the exact point of intersection

5m