Differentiation : Chrastmas 2012

Q1(a) $\frac{dy}{dx} = \boxed{2x - 6}$ (general Rule) 5m

(b)(i) $F(x) = 5 - 3x$

① $F(x+h) = 5 - 3(x+h) = 5 - 3x - 3h$

② $F(x+h) - F(x) = (5 - 3x - 3h) - (5 - 3x)$

$= -3h$

Q1 (60)

③ $\frac{F(x+h) - F(x)}{h} = -3$

④ $\lim_{h \to 0} = -3$ Ans $\boxed{-3}$ 10m

(ii) $y = \underset{u}{(x^2 - 4)}\underset{v}{(3x - 1)}$

$u\frac{dv}{dx} + v\frac{du}{dx}$

$u = x^2 - 4 \quad \frac{du}{dx} = 2x$

$v = 3x - 1 \quad \frac{dv}{dx} = 3$

$= (x^2 - 4)(3) + (3x - 1)(2x)$ @ $x = 2$ 7m

$= ((2)^2 - 4)(3) + (3(2) - 1)(2(2))$

$= (0)(3) + (5)(4)$

$= \boxed{20}$ 3m

(c) Speed $= 96 + 40t - 4t^2$

(i) speed $= 96$

$96 + 40t - 4t^2 = 96$

$4t^2 - 40t + 96 - 96 = 0$

$4t^2 - 40t = 0 \quad \div 4$

$t^2 - 10t = 0$

$t(t - 10) = 0$ 5m

$t = 0 \quad t = 10$

at $\boxed{0 \text{ seconds}}$ & $\boxed{10 \text{ Seconds.}}$

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(c)(ii) acceleration → differentiate speed.

$\frac{dv}{dt} = 40 - 8t$ @ $t = 2.5$

$40 - 8(2.5)$

$40 - 20 = \boxed{20\ m/s^2}$ 5m

(iii) acceleration is negative $\Rightarrow < 0$

$40 - 8t < 0$

$40 < 8t$

$5 < t$

$\boxed{t > 5}$ 5m

(d)(i) $F(-3) = (-3)^3 - 3(-3) + 1 = -17$ $(-3, -17)$

$F(-2) = (-2)^3 - 3(-2) + 1 = -1$ $(-2, -1)$

$F(0) = (0)^3 - 3(0) + 1 = 1$ $(0, 1)$

$F(2) = (2)^3 - 3(2) + 1 = 3$ $(2, 3)$

$F(3) = (3)^3 - 3(3) + 1 = 19$ $(3, 19)$ 5m

(ii) $f(x) = x^3 - 3x + 1$

$F'(x) = 3x^2 - 3$ 5m

(iii) @ max / min $\frac{dy}{dx} = 0$.

$3x^2 - 3 = 0$ $(\div 3)$

$x^2 - 1 = 0$ (perfect Squares)

$(x+1)(x-1) = 0$

$x = -1 \quad x = 1$ 5m

$x = -1$

$y = x^3 - 3x + 1$

$y = (-1)^3 - 3(-1) + 1$

$y = 3$

$(-1, 3)$ Max

$x = 1$

$y = (1)^3 - 3(1) + 1$

$y = -1$

$(1, -1)$ Min 5m

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Q2 (35)

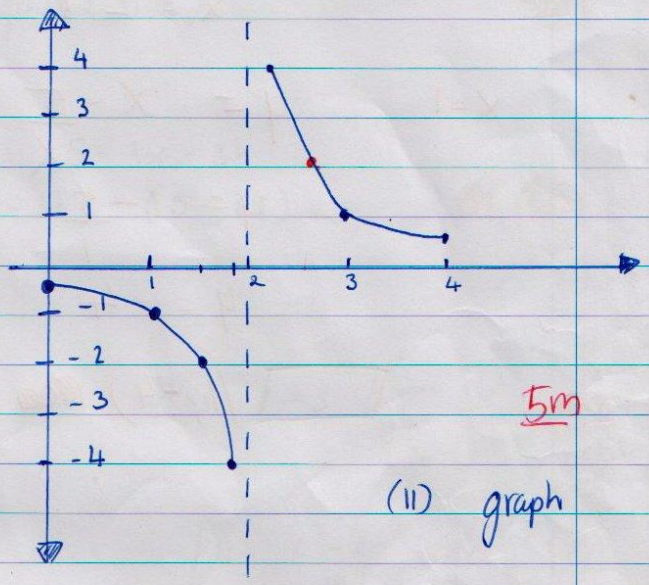
Q2. $h(x) = x^2 + 1$ 10m

(a) $h(x) = 50$

$x^2 + 1 = 50$ $x^2 = 49$ $x = \sqrt{49}$ $x = 7$ (OR $x = -7$)

(b) (i)

$x = 0$	$y = -1/2$
$x = 1$	$y = -1$
$x = 1.5$	$y = -2$
$x = 1.75$	$y = -4$
$x = 2.25$	$y = 4$
$x = 3$	$y = 1$
$x = 4$	$y = 1/2$
$x = 2.5$	$y = 2$ 5m



5m

(ii) graph

(c)(i) $F(x) = \frac{x - 5}{x}$ Rewrite to avoid $\frac{u}{v}$ Rule

$F(x) = x - 5x^{-1}$

$\boxed{F'(x) = 1 + 5x^{-2}}$ 5m

(ii) parallel means same slopes.

Slope of $y = 6x$ is $\frac{dy}{dx} = 6$.

So $\frac{dy}{dx}$ $(F'(x))$ is 6. 5m

$1 + 5x^{-2} = 6$

$\frac{1}{1} + \frac{5}{x^2} = \frac{6}{1}$

$x^2 + 5 = 6x^2$

$x^2 - 6x^2 + 5 = 0$

$(x-1)(x-5) = 0$

$\boxed{x = 1}$ $\boxed{x = 5}$

$y = \frac{x-5}{x}$ $y = \frac{5-5}{5}$

$y = 1 - 5$ $\boxed{y = 4}$

$\boxed{y = -4}$

5m

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Q3. Complex Numbers

Q3 Total 60

(i) $(3+2i)(2-5i)$ (split)

$3(2-5i)+2i(2-5i)$

$6-15i+4i-10i^2$

$6-11i-10(-1)$

$10+6-11i$

$z_1 = \boxed{16-11i}$ 5m

(ii) $(5+4i)(17-13i)$

$5(17-13i)+4i(17-13i)$

$85-65i+68i-52i^2$

$85+3i-52(-1)$

$\boxed{137+3i}$

$(5+3i)(17-13i)$

$5(17-13i)+3i(17-13i)$

$85-65i+51i-39i^2$

$85-14i-39(-1)$

$\boxed{124-14i}$

$(137+3i)-(124-14i)$

~~$40-33i$~~ 10m

$\boxed{13+17i}$

(iii) $\left[\frac{5}{2}+\frac{7}{2}i\right]\left[\frac{5}{2}+\frac{7}{2}i\right]$

$\frac{5}{2}\left[\frac{5}{2}+\frac{7}{2}i\right]+\frac{7}{2}i\left[\frac{5}{2}+\frac{7}{2}i\right]$

$\frac{25}{4}+\frac{35}{4}i+\frac{35}{4}i+\frac{49}{4}i^2$

$\frac{25}{4}+\frac{70}{4}i-\frac{49}{4}$

$\boxed{-\frac{24}{4}+\frac{70}{4}i}$

$\left[\frac{5}{2}+\frac{1}{2}i\right]\left[\frac{5}{2}+\frac{1}{2}i\right]$

$\frac{5}{2}\left[\frac{5}{2}+\frac{1}{2}i\right]+\frac{1}{2}i\left[\frac{5}{2}+\frac{1}{2}i\right]$

$\frac{25}{4}+\frac{5}{4}i+\frac{5}{4}i+\frac{1}{4}i^2$

$\frac{25}{4}+\frac{10}{4}i-\frac{1}{4} = \boxed{\frac{24}{4}+\frac{10}{4}i}$

Subtract 2 Results.

$\left(-\frac{24}{4}+\frac{70}{4}i\right)-\left(\frac{24}{4}+\frac{10}{4}i\right)$

$-\frac{48}{4}+\frac{60}{4}i$

$\boxed{-12+15i}$ Answer 10m

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(iv) $z_4 = 1 + i + i^2 + i^3$

$z_4 = 1 + i + (-1) + (-i)$

$z_4 = 1 - 1 + i - i$

$\boxed{z_4 = 0}$ 5/

* $\boxed{i^3 = (i^2)(i) = -i}$

(b) $z_1 = 16 - 11i = \sqrt{a^2 + b^2} = \sqrt{(16^2) + (11)^2} = \sqrt{377} = 19.4$

$z_2 = 13 + 17i = \sqrt{a^2 + b^2} = \sqrt{(13)^2 + (17)^2} = \sqrt{458} = 21.4$

z_2 is furthest @ 21.4 5/

(c)(i) $\sqrt{-144} = \sqrt{144}\sqrt{-1} = 12i$ 5/

$i^{65} = (i^2)^{32}(i) = (-1)^{32}(i) = i$ 5/

(ii) $(1+3i)(i) = i + 3i^2 = -3 + i$

(i) $(2.5+3i)(i) = 2.5i + 3i^2 = -3 + 2.5i$

$(1 + 1.5i)(i) = i + 1.5i^2 = -1.5 + i$

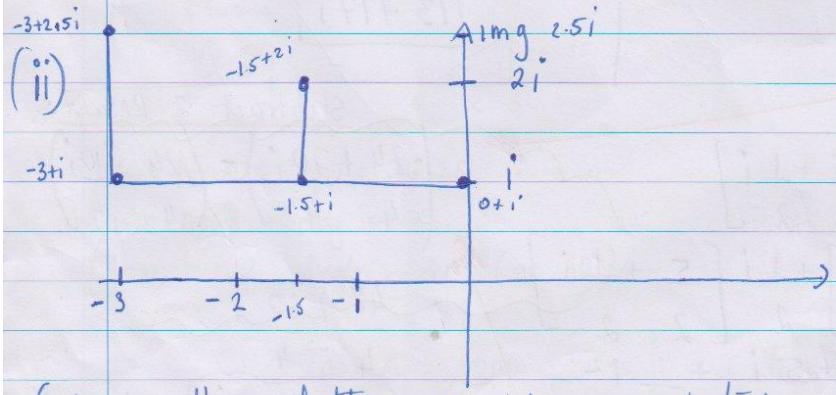
$(2 + 1.5i)(i) = 2i + 1.5i^2 = -1.5 + 2i$

$(1 + 0i)(i) = i + 0i^2 = 0 + i$

Remember $\boxed{i^2 = -1}$

5/

(ii)



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(iii) the letter F has rotated 90° anticlockwise

5/

Q4 (30)

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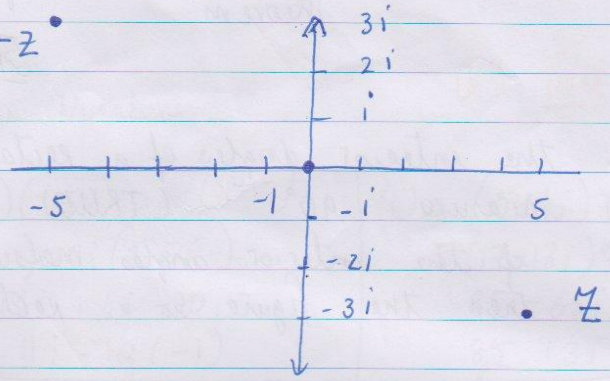
Q4 $Z = 5 - 3i$

(i) $-Z = -(5-3i) = -5+3i$ 5/



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(ii) $= |Z - 1|$

$= |5 - 3i - 1|$

$= |4 - 3i|$ 5/

$\sqrt{a^2+b^2} = \sqrt{(4)^2+(-3)^2} = \sqrt{16+9} = 5$

(iii) $K(Z + \bar{Z}) = 24$

$K(6-4i+6+4i) = 24$

$K(12) = 24$

$K = 24 \div 12$

$\boxed{K = 2}$

$\bar{Z} = 6+4i$

$Z = 6-4i$

5/

(iv) $\frac{S+ti}{4+3i} = Z$

$\frac{S+ti}{4+3i} = \frac{6-4i}{1}$

$S+ti = (6-4i)(4+3i)$

$S+ti = 24+18i-16i-12i^2$

$S+ti = 24+2i+12$

$S+ti = 36+2i$

Real: $\boxed{S = 36}$

Img: $ti = 2i$, $\boxed{t = 2}$

10m

Q5. Converse of a theorem: Its the opposite or Reverse statement of the theorem

5m

(b)(i) Statement: the interior angles of a rectangle each measure 90° (TRUE) 5m

Converse: if the interior angles measure 90°, then the figure is a rectangle (FALSE) 5m

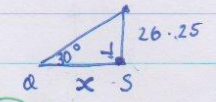
MORE (Statement: if a shape is a square then the angles add up to 360° (TRUE)

Converse: If the angles in a shape add up to 360° then its a SQUARE (FALSE)

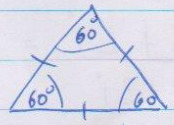
(b)(ii) Scalene Triangle: no equal angles or Sides 5m

(c)(i) $|ST| =$ Radius $\times$ 8 = Diameter $\times$ 4 = $52.5 \times 4 = 210$

$|QS| =$



5m



equilateral △

we have opp = 26.25
we need adj = x.
we have Angle < 30° (or 90°)

TAN A = $\frac{\text{opp}}{\text{adj}}$

$\text{Tan } 30° = \frac{26.25}{x}$

(OR Sine Rule)

$x \text{ Tan } 30° = 26.25$

$x = (26.25) \div (\text{Tan } 30°)$

$x = 45.466 = 45.47\text{mm}$

5m

Q5. (35)

(c) (ii) length of one side of Triangle is:

$|ST| + |QS| + |TR|$

$= 210 + 45.466 + 45.466$

$= 300.932$ (one side)

$= 902.796$ (3 Sides of Triangle)

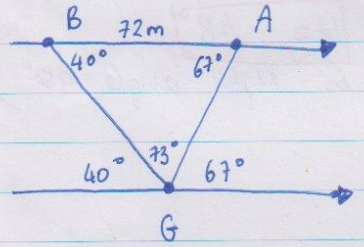
if 10% of 902.796 will be lost (wastage)

10% $= 90.2796$ (is wasted) 5m

$902.796 + 90.2796$

$\boxed{993.08}$

Q6 (a)



- River banks are parallel
- $\angle$ alternate angles

(i) G to A

$\frac{x}{\sin 40^\circ} = \frac{72}{\sin 73^\circ}$

$x \sin 73^\circ = 72 \sin 40^\circ$

$x = \frac{72 \sin 40^\circ}{\sin 73^\circ}$

$x = 48.395$ 5m

$|GA| = 48.395$

G to B

$\frac{x}{\sin 67^\circ} = \frac{72}{\sin 73^\circ}$

$x \sin 73^\circ = 72 \sin 67^\circ$

$x = \frac{72 \sin 67^\circ}{\sin 73^\circ}$

$x = 69.304$

(ii) $|GB| = 69.304$ 5m

Q6 (40)

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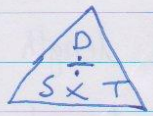
(b)(i) Time = $\frac{D}{S}$ = $\frac{48.395}{0.9}$ = 53.77 seconds G to A



5m

Time = $\frac{D}{S}$ = $\frac{69.304}{3.2}$ = 21.657 seconds G to B

5m

(c) Lots of possible — Logical / Common Sense Reasons

Any
- Current is too dangerous
- She might be parked closer to point A
- getting out at point B might be too Difficult

5m

(d) X = 90° (angle in semi circle on diameter) 5m

y = (90 + 42) − (180)° y = 48° 5m

Z = 42° (Z + y is Right angle 90°) 5m

787.5 3 Sides 10% 78.75 866.25