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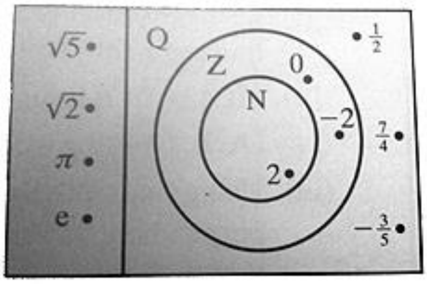
Complex Numbers Notes Leaving Cert Higher



Irrational Number:

Any real number that can't be expressed in form $\frac{a}{b}$, where a and b are integers.

Note: b can't be zero examples of rational numbers are √5, ∏ ≈ 3.141592654……

Constructions: page 94 + 95 should be known

Adding / Subtracting Complex Numbers:

Add real to real and imaginary to imaginary (3 + 2i) + (1 - 5i) = 4 -3i

Multiplying Complex Numbers:

(3 + 5i) (4 – 3i) = 3 (4 - 3i) + 5i (4 – 3i) = 27 + 11i

Split brackets and multiply remember $i^2 = -1$ also $i^3 = -i$ $i^4 = 1$ $i^5 = i$ etc

Dividing Complex Numbers: Use if there is an 'i' in the denominator

Use the conjugate rule to multiply numerator and denominator

$$\frac{3+4i}{2-5i} = \frac{3+4i}{2-5i} \times \frac{2+5i}{2+5i} = -\frac{14}{29} + \frac{23}{29}i$$

Solving equations:

Eg: Solve x^2 +2 x + 2 = 0 use the – b formula $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

Note $i = \sqrt{-1} = i$ $\sqrt{-16} = 4i$ $\sqrt{-4} = 2i$ etc

Argand Diagram: is the graph of a complex number with a real and imaginary axis

Modulus: $\sqrt{(\mathbf{a})^2 + (\mathbf{b})^2}$ this finds the distance of a complex number from the origin

$|-2 + 2i| = \sqrt{(-2)^2 + (2)^2} = \sqrt{8}$

Conjugate Roots Theorem:

The roots (i.e the solution to the –b formula) of a quadratic equation occur in conjugate pairs if all the coefficients are **REAL**

$Z^2 - 4Z + 5 = 0$ has the roots 2 + i and 2 – i **(a conjugate pair)** because the coefficients 1, -4 and 5 are all **REAL**

To form a quadratic equation when given the roots we use

$\boldsymbol{Z^2}$ **- (sum of roots) $\boldsymbol{Z}$ + (product of roots) = 0**

Cartesian Form: is a complex number written in form *x + yi*

Polar Form: is a complex number is an equation written in form $\mathbf{r}\,(\cos\theta + i\sin\theta)$
Where **r is the modulus** length and θ **is the angle** the complex number makes with the positive x axis.

See page 118/119 in your text book 6

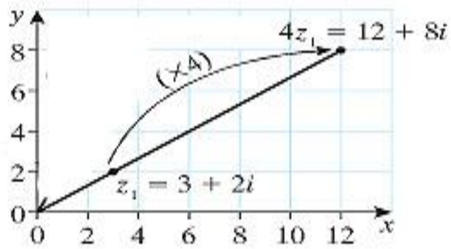
*We use Tan $^{-1}$ (2nd function on the calculator) to work out the angle and give our angle θ in radian mode **not** degrees remember **Π =** 180 degrees

Transformations of Complex Numbers:

1. Multiplying a complex number by a real number

If a complex number $z_1 = 3 + 2i$ is multiplied by 4, we get $4z_1 = 4(3 + 2i) = 12 + 8i$.
The real part is increased by a factor of 4 and the imaginary part is also increased by a factor of 4.

The complex number appears to be **stretched** along a line from the origin by a factor of 4.



2. Multiplying by i

When a complex number such as $4 + i$ is multiplied by i, the complex number **rotates anti-clockwise about the origin by a quarter of a turn.**

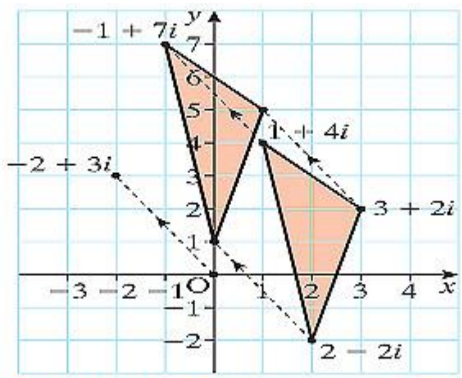
For example, $(4 + i).i = 4i + i^2$
$= 4i - 1$
$= -1 + 4i$
... a rotation of 90°

$z \times i$, z rotates by 90°
$z \times (i)^2$, z rotates by 180°
$z \times (i)^3$, z rotates by 270°
$z \times (i)^4$, z rotates by 360°
$z \times (-i)$, z rotates by $(-90°)$

3. Adding complex numbers

(i) When a complex number z is added separately to other complex numbers – z_1, z_2, z_3 – it creates a **translation of the plane.**

Let $z = -2 + 3i$
and $z_1 = 3 + 2i$, $z_2 = 1 + 4i$, $z_3 = 2 - 2i$.
Then $z + z_1 = -2 + 3i + 3 + 2i = 1 + 5i$
$z + z_2 = -2 + 3i + 1 + 4i = -1 + 7i$
$z + z_3 = -2 + 3i + 2 - 2i = i$



Translation mapping $z_1 \longrightarrow z_1 + z$