Definitions and Theorem Statements

Converse – this means the opposite or the reverse of a theorem.

Corollary – this is a statement attached to a Theorem which has been proven and follows obviously from it.

Axiom – is a statement accepted **without** proof, e.g. Angles in a straight line add up to 180 degrees.

Theorem – is a statement that can be shown to be true, through the use of axioms and logical argument.

Transversal – a line which crosses over parallel lines.

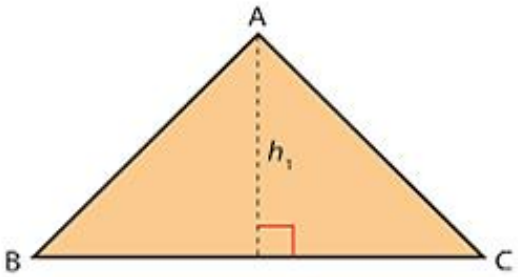
Similar – Two triangles are similar if 2 angles in one are equal to 2 angles in the other **(note: the sides don't have to be equal in length)**

Triangle Theorems

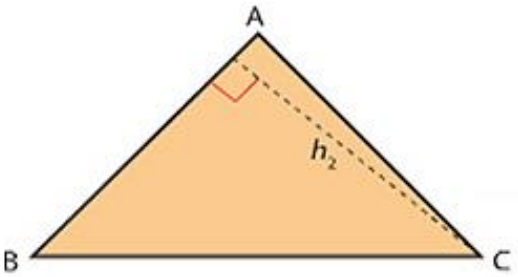
Area Formula for a triangle – ½ the base multiplied by the perpendicular height

Theorem – For any triangle, base multiplied by height does not depend on the choice of base.

For example, Given 2 identical triangles, and choosing different sides as bases, thus different perpendicular heights, the area will still be the same.



$$\text{Area} = \frac{1}{2}|BC| \times h_1$$



$$\text{Area} = \frac{1}{2}|AB| \times h_2$$

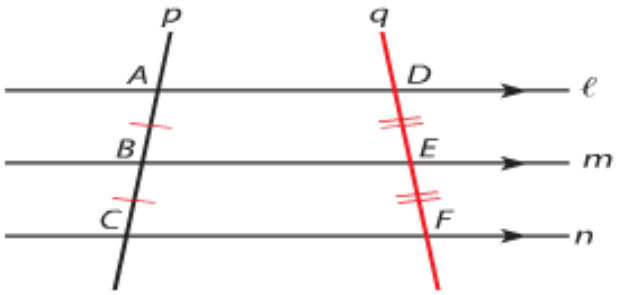
Theorem - The angle opposite the longer of 2 sides is greater than the angle opposite the shorter side.

The **converse** of this theorem is also **true**; the side opposite the larger of 2 angles is greater than the side opposite the smaller angle.

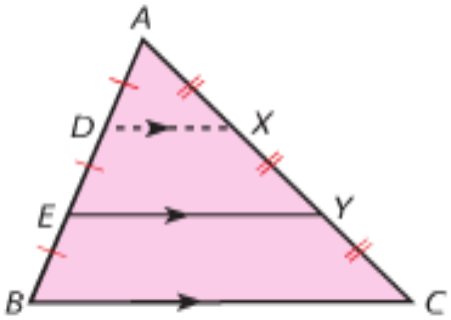
[Example of a **FALSE** converse:
In a rectangle all angles are 90 degrees.
Converse: If in a figure all angles are 90 degrees then it's a rectangle (false)

Theorem – Two sides of a triangle added together are greater than the third side.

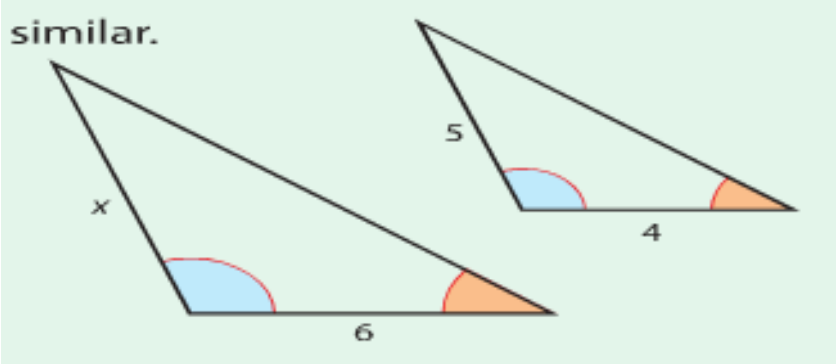
Theorem – If three parallel lines cut off equal segments on some transversal then they will cut off equal segments on any transversal.



Theorem – A line drawn parallel to the side of a triangle divides the other two sides in the same ratio.



Theorem – If two triangles ABC and DEF are similar (same size angles), then their sides are proportional in order,



$$\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$$

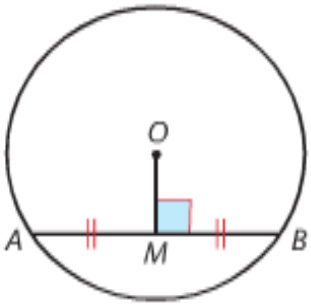
Parallelograms

Theorem for Area of a parallelogram – base multiplied by perpendicular height.

Theorem – A diagonal of a parallelogram bisects the area.

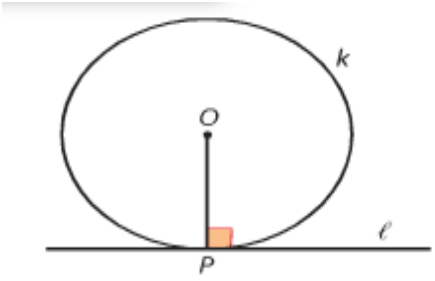
Circle Theorems

Theorem – The perpendicular from the centre of a circle to a chord bisects the chord.

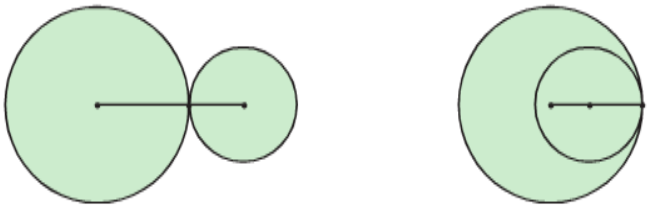


Theorem – A tangent is perpendicular to the radius that touches the point of contact.

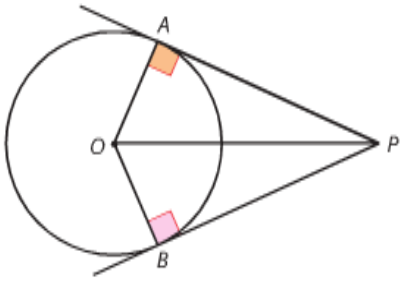
Theorem –If a point ***P***, lies on the circle ***k***, and a line ***l*** is perpendicular to the radius at ***P***, then ***l*** is a tangent to ***k***.



Corollary – If two circles intersect at one point only, then the two centres and the point of contact are collinear



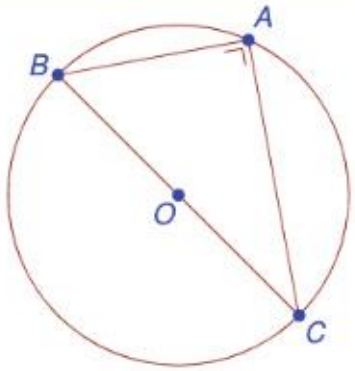
The lengths of 2 tangents from a point to the circle are equal.



Some further Circle results that you should know:

(i) Top angle in a semicircle (where base is diameter) is always 90 degrees

Each angle in a semicircle is a right angle.



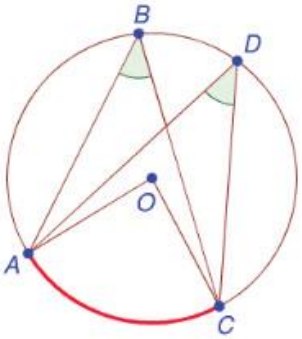
(ii) Two angles standing on the same Arc are equal

Two Angles in a Circle Standing on the Same Arc

In the diagram, both angles at *B* and *D* are on the same arc, *AC*.

Corollary 2

All angles at points of a circle, standing on the same arc, are equal.

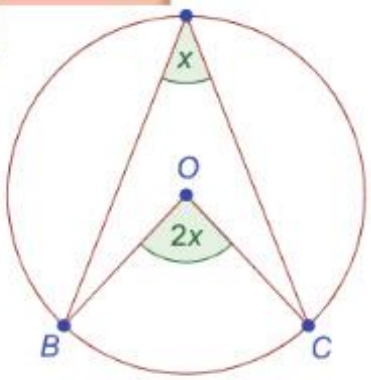


(iii) Angle at centre is twice the angle at the top on the same Arc

The angle at the centre of the circle is twice the measure of the angle at the circumference.

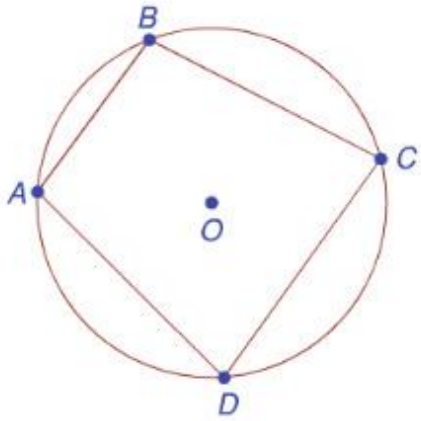
FORMULA

$|\angle BOC| = 2|\angle BAC|$



(iv) Opposite angles in a cyclic quadrilateral add to 180 degrees

Cyclic Quadrilaterals



A quadrilateral in which all four vertices (corners) are points on the circle is referred to as a **cyclic quadrilateral**.

The **opposite angles** in a cyclic quadrilateral add up to 180 .