Summer 2014 Solution

Q1a $(\sqrt{3x+13})^2 = (\sqrt{x}+3)^2$

$3x+13 = \sqrt{x}(\sqrt{x}+3)+3(\sqrt{x}+3)$

$3x+13 = x+6\sqrt{x}+9$

$3x+13-x-9 = 6\sqrt{x}$

(square again) $2x+4 = 6\sqrt{x}$

$(2x+4)^2 = (6\sqrt{x})^2$

$4x^2+16x+16 = 36x$

$4x^2-20x+16=0$

$x^2-5x+4=0$

$x=1 \quad x=4$

Test in original

$x=1$

$\sqrt{3(1)+13} = (\sqrt{1}+3) \quad 4=4$ true

$x=4$

$\sqrt{3(4)+13} = \sqrt{4}+3 \quad 5=5$ true

$\boxed{x=1 \quad x=4}$

(0, 3, 5, 8, 10)

(b)(i) $\frac{x+3}{x-4} < 2$

$(x-4)^2 \frac{(x+3)}{(x-4)} < 2\frac{(x-4)^2}{1}$

multiply $(x-4)^2$ as its +ve and we can keep arrow as <

$(x-4)(x+3) < 2(x^2-8x+16)$

$x^2-x-12 < 2x^2-16x+32$

$0 < x^2-15x+44$

$x^2-15x+44>0$

$x=4 \quad x=11$

$4 > x > 11$

Test $x=7$ for eg

$\frac{7+3}{7-4} < 2$

$\frac{10}{3} < 2$

$\Leftarrow$ $3\frac{1}{3} < 2$ false

(0, 3, 5, 8, 10)

(ii) Real roots $b^2-4ac \geq 0$

$(2p+1)(x^2)+(p+2)(x)+1=0$

$a = 2p+1 \quad b = p+2 \quad c=1$

$(p+2)^2-4(2p+1)(1) \geq 0$

$p^2+4p+4-8p-4 \geq 0$

$p^2-4p \geq 0$

$p(p-4) \geq 0$

$p \geq 0 \quad p \geq 4$

(0, 3, 7, 10)

(i) $t=0$ $Q(t) = 2.920$

$Q(t) = Ae^{-bt}$

$2.920 = Ae^{-b(0)}$

$2.920 = Ae^{0}$

$\boxed{2.920 = A}$

$t=1$ $Q(t) = 2.642$

$2.642 = 2.920\,e^{-b(1)}$

$\frac{2.642}{2.920} = e^{-b}$

$0.90479 = e^{-b}$

$\ln 0.90479 = \ln e^{-b}$ $(\ln e) = 1$

$\ln 0.90479 = -b\,(1)$

$-0.10005 = -b$

$0.10005 = b$

$\boxed{0.100 = b}$

5M

(0,3,5)

(ii) $t=2$ $Q(t) = 2.391$

$2.391 = (2.920)\,e^{-(0.100)(2)}$

$2.391 = 2.920\,e^{-0.2}$ (from Calculator)

$2.391 = 2.391$

therefore $A = 2.920$, $b = 0.100$ are verified

(0,3,5) 5m

Complex Numbers

Q2 $\boxed{\frac{T_2}{T_1} = \frac{T_3}{T_2}}$ method

$\frac{6+4i}{2-3i} = \frac{-8+pi}{6+4i}$

$(2-3i)(-8+pi) = (6+4i)(6+4i)$

$-16+2pi+24i-3pi^2 = 36+48i+16i^2$ $i^2=-1$

$-16+3p+i(2p+24) = 20+48i$

Let LHS = RHS

Real $-16+3p = 20$

$3p = 36$

$p = 12$

Note: No need to do img part as we now have $p=12$.

(0,3,7,10)M

(i)

z_1	z_2	z_3	z_4	z_5	z_6
$\sqrt{3}+i$	$0+2i$	$-\sqrt{3}+i$	$-\sqrt{3}-i$	$0-2i$	$\sqrt{3}-i$

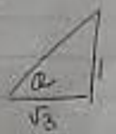
(ii) $R(\cos\theta + i\sin\theta)$

$z_1 = \sqrt{3}+i$

$R = \sqrt{(\sqrt{3})^2 + (1)^2}$

$R = \sqrt{4}$

$R = 2$

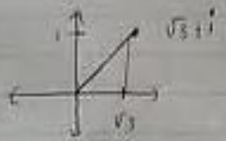


$\tan\theta = \frac{O}{A}$

$\tan\theta = \frac{1}{\sqrt{3}}$

$\tan^{-1} 1/\sqrt{3} = 30° \,/\, \frac{\pi}{6}$

(0, 2, 4, 5) M



$R(\cos\theta + i\sin\theta)$

$\boxed{2\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)}$

(iii) $z^6 = a+bi$

De Moivres
$R^n(\cos n\theta + i\sin n\theta)$

$2^6\left(\cos\frac{6\pi}{6} + i\sin\frac{6\pi}{6}\right)$

$64(-1+0i) = -64+0i \Rightarrow$ $a=-64$, $b=0$

$a+bi$

(0, 2, 5) M

Q3 The line AB $\quad A(2,2) \quad B(6,-6)$ ($x_1 y_1$, $x_2 y_2$)

(a)

$y-2 = -2(x-2)$

$y-2 = -2x+4$

$\boxed{2x+y-6=0}$

$M = \frac{-6-2}{6-2} = \frac{-8}{4} = -2$

5m (0, 3, 5)

(b) $x=0$ on y axis $\quad$ ~~D (0,6)~~

$2(0)+y-6=0$

$y=6$

5m (0, 3, 5)

(c) $\left|\frac{ax_1 + by_1 + c}{\sqrt{a^2+b^2}}\right|$

$2x + y - 6 = 0$

$a = 2 \quad b = 1 \quad c = -6$

$C = (-2, -3)$ $x_1 \; y_1$

OR $\frac{13\sqrt{5}}{5}$

$\left|\frac{(2)(-2) + (1)(-3) - 6}{\sqrt{(2)^2 + (1)^2}}\right| = \left|\frac{-13}{\sqrt{5}}\right| = \frac{13}{\sqrt{5}}$

10m (0, 3, 5, 10)

(d) Area ADC.

$x_1 y_1 \quad x_2 y_2$

$|AD| = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$ $\quad A = (2,2) \quad D(0,6)$

$\sqrt{(0-2)^2 + (6-2)^2}$

$\sqrt{20}$

$2\sqrt{5}$

5m (0, 3, 5)

Area of ADC $= \frac{1}{2}$ (base) $\times \perp$ height

$= \frac{1}{2}(2\sqrt{5})\left(\frac{13}{\sqrt{5}}\right)$

$= 13$

* Hence means we must you result from (c)

Circle

Q4. $C_1 = x^2 + y^2 - 2x - 4y - 20 = 0$

(a) $x^2 + y^2 + 2gx + 2fy + c = 0$

$2g = -2 \quad 2f = -4$ $\quad$ Radius $= \sqrt{g^2 + f^2 - c}$

$-g = 1 \quad -f = 2$ $\quad = \sqrt{(-1)^2 + (-2)^2 - (-20)}$

$(-g, -f)$ $\quad = \sqrt{25}$

$\boxed{\text{Centre } (1,2)}$ $\quad$ $\boxed{\text{Radius} = 5}$ $\quad$ $\boxed{0, 2, 5}$ m.

(b)(i) $\left|\frac{ax_1 + by_1 + c}{\sqrt{a^2+b^2}}\right|$ $\quad 2x - y + 5 = 0$ $\quad a = 2 \quad b = -1 \quad c = 5$

Centre $(1,2)$ $x_1 y_1$

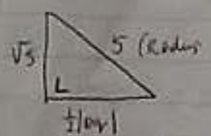
$\left|\frac{2(1) - 1(2) + 5}{\sqrt{(2)^2 + (-1)^2}}\right|$

$\left|\frac{5}{\sqrt{5}}\right| = \boxed{\sqrt{5}}$ $\quad$ $\boxed{0, 2, 5}$ m

(ii) |PQ|

method
pythagoras

(perp distance)



$(5)^2 = (\sqrt{5})^2 + \left(\frac{|PT|}{2}\right)^2$

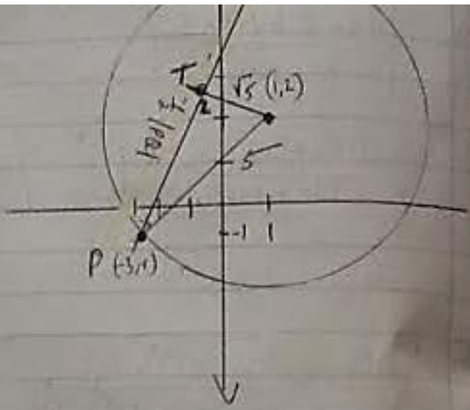
$20 = |PT|^2$

$\sqrt{20} = |PT|$

$2\sqrt{5} = PT$

$|PQ| = 2 \times 2\sqrt{5}$

$|PQ| = 4\sqrt{5}$ (0,2,4,5) M

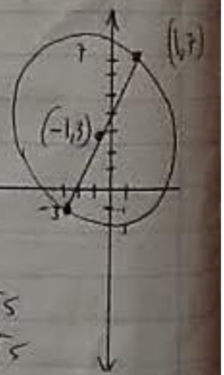


OR find where circle & line intersect using
Rearrange & Sub in
P(-3,-1)   Q(1,7)   and then use distance formula = $4\sqrt{5}$

(c). $|PQ| = 4\sqrt{5}$ $P(\underset{x_1}{-3}, \underset{y_1}{-1})$ $Q(\underset{x_2}{1}, \underset{y_2}{7})$

Midpoint of PQ = Centre

$\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right) = \left(\frac{-3+1}{2}, \frac{-1+7}{2}\right) = (-1, 3)$
h k



$(x-h)^2 + (y-k)^2 = R^2$

$(x+1)^2 + (y-3)^2 = (2\sqrt{5})^2$

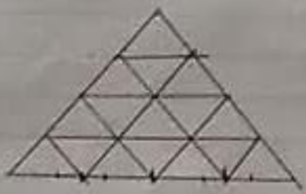
$\boxed{(x+1)^2 + (y-3)^2 = 20}$

diameter $PQ = 4\sqrt{5}$
Radius $= 2\sqrt{5}$

(0,3,5,8,10) M

patterns / sequences

Q5



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(ii)

Pattern	1^{st}	2^{nd}	3^{rd}	4^{th}
no of small Δs	1	4	9	16
no of matchsticks	3	9	18	30

5

(b)

	~~[illegible]~~	1, 4, 9, 16
a	1st diff	3 5 7
2a	2nd diff	2 2

× not linear arithmetic

✓ quadratic

10/

$T_n = an^2 + bn + c$

$T_n = 1n^2 + bn + c$

$T_1 = 1$

$T_1 = (1)^2 + b(1) + c = 1$

$\boxed{b + c = 0}$

$T_2 = 4$

$(2)^2 + b(2) + c = 4$

$\boxed{2b + c = 0}$

$2a = 2$

$a = 1$

$(1, 1)$

$b + c = 0$

$\ominus\ 2b + c = 0$

$-b = 0$

$\boxed{b = 0}$

$0 + c = 0$

$\boxed{c = 0}$

$T_n = n^2 + bn + c$

$T_n = n^2 + 0(n) + 0$

$\boxed{T_n = n^2}$

$\boxed{0, 3, 5, 8, 10}$

or you can get by observation of

$T_1 \quad T_2 \quad T_3 \quad T_4 \quad T_n$

1, 4, 9, 16

$(1)^2, (2)^2, (3)^2, (4)^2, (n)^2$

(c) pattern :

	T_1	T_2	T_3	T_4
pattern	3,	9,	18,	30
1^{st} diff		6	9	12
✓ 2^{nd} diff			3	3

not arithmetic
not geometric

quadratic.

$2a = 3$

$a = \frac{3}{2}$

$T_n = an^2 + bn + c$

$= \frac{3}{2}n^2 + bn + c.$

$T_1 = 3 \quad \frac{3}{2}(1)^2 + b(1) + c = 3$

$b + c = 1\frac{1}{2}$

$T_2 = 9 \quad \frac{3}{2}(2)^2 + b(2) + c = 9.$

$2b + c = 3.$

$b + \cancel{c} = 1\frac{1}{2}$

$\ominus \; 2b + \cancel{c} = 3$

$-b = -1\frac{1}{2}$

$\boxed{b = 1\frac{1}{2}}$

$b + c = 1\frac{1}{2}$

$c = 1\frac{1}{2} - b$

$c = 1\frac{1}{2} - 1\frac{1}{2}$

$\boxed{c = 0}$

$\boxed{T_n = \frac{3}{2}n^2 + \frac{3}{2}n + 0}$

10/

(0, 3, 5, 8, 10)

$T_{n-1} = \frac{3}{2}(n-1)^2 + \frac{3}{2}(n-1)$

$= \frac{3}{2}(n^2 - 2n + 1) + \frac{3n}{2} - \frac{3}{2}$

$= \frac{3}{2}n^2 - \cancel{3n} + \cancel{\frac{3}{2}} + \cancel{\frac{3n}{2}} - \cancel{\frac{3}{2}}$

$= \frac{3}{2}n^2 - \frac{3}{2}n$

$(T_n) - (T_{n-1}) = \left(\cancel{\frac{3}{2}n^2} + \frac{3}{2}n\right) - \left(\cancel{\frac{3}{2}n^2} - \frac{3}{2}n\right)$

$= \frac{3}{2}n + \frac{3}{2}n$

$= 3n$

(d) $U_n = an^2 + bn$ (quadratic)

from (c) $T_n = \left(\frac{3}{2}\right)n^2 + \frac{3}{2}n$

$\Rightarrow \boxed{a = \frac{3}{2}}$ $\boxed{b = \frac{3}{2}}$ 10/

(e) 4134

$T_n = 4134 \quad n = ?$

$\frac{3}{2}n^2 + \frac{3}{2}n = 4134 \quad \times 2$ 10/

$3n^2 + 3n = 8268 \quad \div 3$

$n^2 + n - 2756 = 0$

$(n+53)(n-52) = 0$

$n = -53 \quad \boxed{n = 52}$.

Term no 52 has 4134 matchsticks

$n^2 =$ no of small triangles

$52^2 = \boxed{2704}$ small triangles

(0, 3, 5, 8, 10)

Finance.

Q6

(a) $(1+R)^{12} = 1 + i$.

$(1+R)^n = 1 + 0.04$

$(1+R)^n = 1.04$

$1+R = \sqrt[12]{1.04}$

$1+R = 1.00327374$

$R = 1.00327374 - 1$

$R = .00327374$ (as decimal

R as %

$\boxed{0.327\%}$

10/

(a)(ii) $15,000 = P(1.00327)^{36} + \ldots P(1.00327)^{1}$

Sum of all these values, P is unknown

$a = P(1.00327)$ $R = 1.00327$ $S_n = 15,000$

$S_n = \frac{a(1-R^n)}{1-R}$ $15000 = \frac{P(1.00327)[1-1.00327^{36}]}{(1-1.00327)}$

$15,000 = \frac{P(-0.1251744)}{-0.00327}$

$15,000 = P(38.279633)$

$391.85 = P$

$\boxed{€392 = P}$

10/

(gave 7/10 for amortisation formula used correctly)

(b) $A = P\frac{i(1+i)^T}{(1+i)^T - 1}$

$P = 15,000$

$T = 36$

$i = 0.00866$ (decimated)

$A = 15,000\left[\frac{0.00866(1+0.00866)^{36}}{(1+0.00866)^{36} - 1}\right]$

$A = 15,000\left[\frac{0.01181219586}{0.3639949028}\right]$

$A = 15,000(0.03245154196)$

5/

$A = €486.77$

$A = \boxed{€487}$

Q7

(a)

$(hyp)^2 = (1)^2 + (1)^2$

$hyp^2 = 2$

$hyp = \sqrt{2}$ 3/

$|ab| = \sqrt{2}$

$|ac| = \sqrt{8} = 2\sqrt{2}$ 3/

3/ (b)(i) Circumcentre intersection $\boxed{\text{Perpendicular Bisectors of Sides}}$

3/ (ii) Incentre " $\boxed{\text{Bisectors of Angles of triangle}}$

Orthocentre: line $\boxed{\text{from vertex drawn perp to other side}}$

3/ (iv) Centroid: " of $\boxed{\text{Medians of triangle}}$

(median - vertex to midpoint opp).

(c)(i) Construction - 5/

(ii) finds centre of gravity 3/

Q8(a) $x+1 =$ factor $x = -1$ root

$f(-1) = (-1)^3 + 5(-1)^2 + k(-1) - 12 = 0$

$-8 - k = 0$

$\boxed{-8 = k}$

$\boxed{0,2,4,5}$ M

(b)
$$
\begin{array}{r|l}
 & x^2 + 4x - 12 \\
\hline
x+1 & x^3 + 5x^2 - 8x - 12 \\
 & -x^3 - 1x^2 \\
\hline
 & 4x^2 - 8x \\
 & -4x^2 - 4x \\
\hline
 & -12x - 12 \\
 & +12x + 12
\end{array}
$$

$x^2 + 4x - 12 = 0$

$(x-2)(x+6)$

$x = 2 \quad x = -6$

$\boxed{(2,0)}\ \boxed{(-6,0)}$

$\boxed{0,3,7,10}$ M

(6) A (1,2)   B(5,2)

(i)

To get A & B points where both graphs are equal $f(x) = g(x)$

$|x-3| = 2$

$x-3 = +2 \qquad x-3 = -2$

$x = 5 \qquad x = 1$

$f(x) = |x-3|$ @ $x = 5$
$y = |5-3|$
$y = 2$
B (5,2)

$f(x) = |x-3|$ @ $x = 1$
$y = |1-3|$
$y = |-2| \qquad y = 2$
A (1,2)

C (3,0)   f(x) cuts x axis $\underline{y=0}$

$|x-3| = 0$

$x-3 = 0$

$\underline{x = 3}$

C(3,0)

D(0,3)   f(x) Cuts y axis $\underline{x=0}$

$y = |x-3|$

$y = |0-3|$

$y = +3$

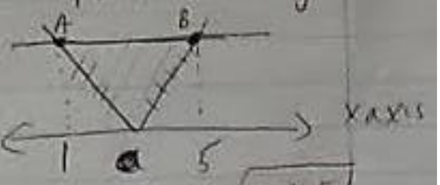
$\boxed{0,3,5}$

A (1,2) B(5,2) C (3,0) D(0,3)

(ii) $|x-3| < 2$ hence

$f(x) < g(x)$ when graph f(x) under g(x)

$1 < x < 5$



$\boxed{0,3,5}$